ASSISTANT SECRETARY OF DEFENSE

3300 DEFENSE PENTAGON
WASHINGTON DC 20301-3300

ECONOMIC
SECURITY

November 23, 1994

MEMORANDUM FOR SECRETARIES OF THE MILITARY DEPARTMENTS
CHAIRMAN OF THE JOINT CHIEFS OF STAFF
UNDER SECRETARIES OF DEFENSE
DIRECTOR, DEFENSE RESEARCH AND ENGINEERING
ASSISTANT SECRETARIES OF DEFENSE
GENERAL COUNSEL OF THE DEPARTMENT OF DEFENSE
INSPECTOR GENERAL OF THE DEPARTMENT OF DEFENSE
DIRECTOR, OPERATIONAL TEST AND EVALUATION
ASSISTANTS TO THE SECRETARY OF DEFENSE
DIRECTOR, ADMINISTRATION AND MANAGEMENT
DIRECTORS OF THE DEFENSE AGENCIES

SUBJECT: 1995 Base Realignments and Closures (BRAC 95) -- Policy Memorandum Two -- Joint Cross-Service Group Functional Analysis Process

This memorandum summarizes the process, involving both Joint Cross-Service Groups (JCSGs) and the individual Military Departments, for developing BRAC alternatives in situations involving such common support functions as labs, depots, test & evaluation, undergraduate pilot training and medical facilities.

JCSGs will determine a functional value for each of the common support functions at each activity within their jurisdiction. These functional values will be independent of the military value of any installation, which is separately determined by the Military Departments. The assessments of functional value and assessments of functional capacity and requirements, using certified data, will then be incorporated into JCSG analyses of possible functional closure or realignment alternatives. The JCSG's (which include representatives from the Military Departments) will use their expertise and judgment to develop these functional closure or realignment alternatives.

To assist them as an analytic tool in this process, the JCSGs will use a linear programming optimization model (documentation attached) to the maximum extent possible. The model provides a basis for further analysis and the application of judgment in developing functional alternatives. While the model has value in assessing alternatives for relocations and consolidations of common support functions, it cannot by itself make recommendations regarding closures or realignments of installations. Those can be made only by the Military Departments or the BRAC 95 Review Group, reflecting judgment concerning the military value of installations, based on the final criteria and the six-year force structure plan.

Each JCSG is currently supported in its evaluations by a Joint Cross-Service Working Group (JCSWG), variously referred to as "sub-groups", "study teams" or "technical and support groups." JCSWGs will adapt the linear programming (optimization) model to assist each JCSG in its analysis and aid in developing alternatives. All JCSGs will be supported by a single Tri-Department BRAC Group consisting of representatives from each Military Department, which will execute runs of the linear programming (optimization) model, using certified data, according to the objective functions and policy imperatives provided by the JCSGs and the management controls required by the internal control plan. JCSG alternatives can be derived from any number of combinations of objective functions and policy imperatives as long as they have been previously approved by the Chairman of the BRAC 95 Steering Group.

The Military Departments will conduct their individual BRAC processes in parallel with the JCSG analyses, to determine the relative military value of their installations. JCSG products such as functional value may be used to assist in determining installation military value. If it is useful to a JCSG in developing its alternatives for analysis, a JCSG may solicit the guidance of the Military Departments concerning the military value of installations. It must be recognized that any such guidance must necessarily be preliminary and will not constitute a final determination of military value or of suitability for closure or realignment.

The JCSGs and the Military Departments will then review the sets of optimization model outputs. Working together, the JCSGs and the Military Departments will apply their collective judgment to develop feasible functional alternatives to facilitate cross-service actions that will strive to maximize infrastructure (overhead) reductions at minimal cost. This cooperative work by the JCSGs and the Military Departments should be completed in time for the BRAC 95 Review Group to consider any issues that may be appropriate and to leave sufficient time for the Military Departments to formulate their recommendations. The JCSGs and Military Departments will continue to interact during November and December as the Military Departments consider cross-service alternatives in their respective BRAC analytical processes.

The Military Departments will present their recommendations for closure and realignment to the Secretary of Defense no later than mid-February, 1995. The Military Departments will provide the Secretary of Defense a status report, to include all preliminary closure and realignment candidates, by January 3, 1995. The Office of the Assistant Secretary of Defense for Economic Security will staff the Military Department recommendations within the Office of the Secretary of Defense. The BRAC 95 Review Group or OSD principals may solicit the opinion of or task the JCSG's during this period, if and as appropriate.

The process described above involves appropriate interaction between JCSG and Military Department analyses and permits consideration of joint functional alternatives to be incorporated within the existing BRAC process of the Military Departments. If you have questions concerning the process, please contact Mr. Robert Bayer, Deputy Assistant Secretary of Defense for Installations, 703-697-1771.

Joshua Gotbaum

Attachment

Joint Cross-Service Analysis Tool User's Guide

Executive Summary

Background

The Deputy Secretary of Defense established policy for the Department of Defense 1995 base realignment and closure (BRAC 95) process with strong emphasis on cross-service opportunities. This document describes operations and capabilities of the common analytical tool to assist Joint Cross-Service Groups (users) in the development of cross-service alternatives as part of the BRAC process.

Analytical Tool

A standard tool often used to develop optimal solutions to complex allocation problems is the mixed-integer, linear program (MILP). The cross-service analysis of allocations of common support functional requirements to Military Department sites and activities is a complex allocation problem.

The MILP formulation described in this document can be used to develop cross-service functional alternatives. The data elements required for this tool are derived from the certified data available to the user. Policy imperatives and other constraints and considerations can be incorporated into the model to allow the tailoring of formulations to accommodate functional attributes and perspectives.

The tool provides the capability to vary the objective function for a formulation in order to obtain families of solutions. A solution defines a set of functional allocations and identification of sites or activities where cross-service functional workload could be assigned. An objective function that combines military value of sites and activities with functional values is discussed in this document. This particular objective function will tend to consolidate common support functions into high military value sites or activities. At the same time, this objective function will assign common support functions to sites having high functional values. The weighting between these two goals can be parameterized to obtain families of solutions for further consideration.

Second and third best alternatives for a given formulation can be obtained using methods described in this document. These alternatives may be considered as additions to the set for further review.

Other objective functions that the user may wish to consider in addition to the one mentioned above, include minimizing excess functional capacity, minimizing the total number of sites performing cross-service functions, and maximizing the sum of functional values. This tool will also allow the user to explore the sensitivity of the optimal solution for a given formulation to particular model inputs.

The MILP formulation described provides the basic analytical tool to generate cross-service functional alternatives.

Contents

User's Guide Organization

This user's guide provides an overview of the analytical methodology in the next section. That section describes the products of the methodology and discusses terminology relating to what a *site or activity* is relative to a *function*.

Section 2 describes the basic data elements that are used in the methodology. Section 2 also discusses data elements in terms of what these elements are meant to represent.

The different optimization problem formulations that the user may choose to use to explore alternatives are discussed in section 3. These include finding a small set of high military value sites or activities that can perform the functional requirement, minimizing excess capacity, and minimizing the number of sites. All of these formulations are parameterized in such a way that the user can explore trade-offs between different factors, such as military value or excess capacity, and assignments of functional requirement based upon functional value. This section also discusses the incorporation of policy imperatives in the optimization problem formulations.

Section 4 demonstrates the application of each of these formulations to a notional set of data. Section 5 describes the methodology for obtaining the second and third best solutions to a given formulation. Finally, section 6 identifies the commercial software product that was used to solve the optimization example problems. Input files for this solver are included in the appendices.

1. Analytical Methodology Overview

The optimization formulations described in this document require a set of data elements as inputs. All of the formulations require a functional value and functional capacity for each site capable of performing that specific cross-service function. The DoD requirement for each cross-service function is needed. Some of the formulations will also require the military values for each site.

A preliminary formulation that allocates cross-service functional requirements based upon functional capacities and functional value will be conducted. The objective function of this formulation will assign the DoD requirement for each cross-service function to sites or activities having the highest functional value for each function. These assignments will only be constrained by the functional capacities at each site. This analysis will not require the military values for the sites.

The primary formulations optimize the assignment of cross-service functions based upon military values of sites, functional values, and capacities. These formulations are very flexible in that multiple objective functions and policy imperatives modeled as constraints may be used to explore different solutions.

A standard resource allocation tool comprises the core of this analytical approach. A standard tool used to find optimal solutions to complex allocation problems is the mixed-integer, linear program (MILP). Allocation of common support functional requirements to military department sites and activities subject to constraints is a complex allocation problem.

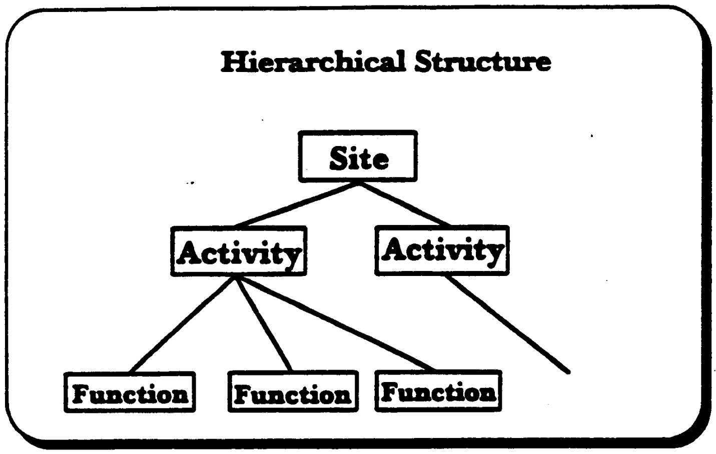
Process Products

The following table lists the various products of the analytical approach defined in this document.

Process products	Description
Capacity analyses	Develop methodology to measure the capacity of a site or activity to perform a function. Use data call responses to calculate capacities.
Requirements analyses	For each function, develop methodology to estimate the out-year DoD requirement to perform the function. Calculate the required capacity and identify excess capacity reduction goals.
Functional value (FV) assessments	Develop measures and weights for assessing the value of performing a function at a site or an activity based upon data call responses. Provide FV for all appropriate functions and site/activity combinations.
Optimize functional requirement allocations (preliminary formulation)	Find the best allocation of functional requirements to sites or activities based solely upon functional capacities and functional values.
Optimize allocations of functional requirements to high military value sites or activities (primary formulations)	Develop solutions based upon the first three products, above, and policy imperatives. Solutions will be developed using the optimization formulations described later in this document as a tool to explore alternatives.

Hierarchical Structure

The Office of the Secretary of Defense (OSD), the departments, and other groups all use different terms to describe the various components of infrastructure that are to be considered by the users. In this document a ***site*** refers to an installation, base, or station. An ***activity*** refers to a component of the site such as depot or test facility residing on the site. A site may have one or more activities. A ***function*** is the capability to perform a particular support action or produce a particular commodity. A common support function is a function. An activity includes a collection of functions. For example, a depot (an activity) may repair engines and airframes. These would be two functions performed at this activity. A function may be further broken down into subfunctions or facilities required to perform functions, but the approach described here does not consider the subfunctions or facilities. Subfunctions or facilities can be incorporated into the process described here if the appropriate data is available. The following diagram illustrates this hierarchical structure.



2. Data Elements

The analytical approach assumes that the following data will be available for all of the sites and functions:

Data Elements	Description
mv_s	Military value of site s expressed as 3 (high), 2 (medium), or 1 (low).
fv_{sf}	Functional value for performing function f at site/activity s expressed as a number from 0 (low) to 100 (high).
cap_{sf}	Capacity of site/activity s to perform function f.
req_f	The total DoD requirement or goal to perform function f.

The military value of a site, mv_s, should measure the overall value of the site.

The fv_{sf} functional value for performing function f at site (or activity) s measures the capability and quality of performing work of type f at site (or activity) s. Capacity to perform a specialized subfunction that is not one of the functions called out in the formulation can be considered in calculating functional value.

3. Optimization Formulations

The mixed integer linear programming (MILP) model formulations, that are described below, serve as the basic analytical tools to assist users in the development of cross-service alternatives, allow for modification of formulations, and incorporation of policy imperatives.[1]

[1]A *policy imperative* is a statement that restricts the solutions that are acceptable and that can be modeled as a constraint in the formulation. An example of a policy imperative is included in one of the examples.

Preliminary Formulation.

The preliminary formulation of the optimization problem will be solved once the initial data (fv_{sf}, cap_{sf}, req_f) are available. This formulation, called **MAXFV** will maximize the functional values weighted by the assigned workload and normalized by the functional requirement. No constraints other than the functional capacities at each site and the requirement to meet the DoD requirement for each cross-service function are included in this formulation. This solution will serve as a baseline of what is possible if no other factors, such as military values of sites or costs, are considered.

For each function, this formulation will load as much of the functional DoD requirement as it can into the site or activity having the highest functional value for that function. If that site or activity does not have the capacity to accommodate the full requirement, the site or activity having the next highest functional value will be allocated any remaining requirement up to its capacity, and so on.

The mathematical description of this formulation follows:

$$\underset{l_{sf}}{Maximize} \ \Sigma_{s\in S} \Sigma_{f\in F} \, l_{sf} \times fv_{sf}/req_f$$

subject to:

$\Sigma_{s\in S} \, l_{sf} = req_f$: for all functions $f \in F$,

$l_{sf} \le k_{sf} \times cap_{sf}$: for all sites $s \in S$ and $f \in F$,

$o_s \le \Sigma_{f\in F} \, k_{sf}$: for all sites $s \in S$,

$k_{sf} \le o_s$: for all sites $s \in S$ and $f \in F$,

$k_{sf} \le \frac{l_{sf}}{\alpha \times cap_{sf}}$: for all functions $f \in F$ and sites $s \in S$,

$0 \le o_s \le 1$, *integer* : for all sites $s \in S$,

$0 \le k_{sf} \le 1$, *integer* : for all sites $s \in S$ and functions $f \in F$;

where

$S =$	The set of all sites under consideration by joint cross-service groups;
$F =$	The set of all functions under consideration by joint cross-service groups;
$o_s =$	1 if any functional requirement is assigned to the site, and 0 otherwise;
$\alpha =$	0.01. No assignment of less than one percent of capacity will be allowed.

Decision variable

$l_{sf} =$	amount of the DoD requirement for function f to be assigned to site s.
$k_{sf} =$	1 if any amount of function f is assigned to site s, 0 otherwise.

The o_s variables are included in this formulation only to keep count of the number of sites that actually have some functional requirement assigned to them. Their inclusion in the model does not affect the assignment of the functional requirement to sites or activities. The two constraints involving the o_s variables are used to ensure that these variables are set to the correct values.

The k_{sf} variables that are structural variables that indicate whether or not any functional workload of type f has been assigned to site s. The α parameter can be used to prevent small functional workload assignments. If α is set to 0.01, then the minimum workload assignment of a function to a site, given that any functional workload for this function is made to this site, would be one percent of that site's capacity to perform that function. The α parameter may be adjusted as required to meet the requirements of the particular user.

Primary Formulations

These formulations explore potential cross-service functional alternatives. The basic formulation is shown below. Specification of the objective function, $f(o_s, l_{tg}, k_{uh})$, will create a different optimization problem.

$$\underset{o_s, l_{tg}, k_{uh}}{Minimize} \; f(o_s, l_{tg}, k_{uh})$$

subject to

$\Sigma_{s \in S} l_{sf} = req_f$: for all functions $f \in F$,

$o_s \leq \Sigma_{f \in F} k_{sf}$: for all sites $s \in S$,

$0 \leq l_{sf} \leq k_{sf} \times cap_{sf}$: for all functions $f \in F$ and sites $s \in S$,

$k_{sf} \leq o_s$: for all sites $s \in S$ and $f \in F$,

$k_{sf} \leq \frac{l_{sf}}{\alpha \times cap_{sf}}$: for all functions $f \in F$ and sites $s \in S$,

$0 \leq o_s \leq 1$, *integer* : for all sites $s \in S$,

$0 \leq k_{sf} \leq 1$, *integer* : for all sites $s \in S$ and functions $f \in F$,

where

$S =$ The set of all sites under consideration by joint cross-service groups;

$F =$ The set of all functions under consideration by joint cross-service groups;

$\alpha =$ 0.01. No assignment of less than one percent of capacity will be allowed.

Decision variables

$o_s =$ 1 if any cross-service functional requirements are assigned to the site or activity, 0 otherwise;

$l_{sf} =$ amount of the DoD requirement for function f to be assigned to site or activity s.

$k_{sf} =$ 1 if any DoD requirement for function f is to be assigned to site s, 0 otherwise.

Three different optimization formulations that vary only in the specification of the objective function are discussed next.

The MINNMV Formulation. This formulation will find a small number of sites having the highest military value that can accommodate the DoD required workload. In addition, it will assign the DoD requirement for each cross-service function to the retained sites (or activities) having the highest functional value for that function. The purpose of this formulation is to assign, to the extent possible, the cross-service functional requirements to sites or activities having high military value and high functional values. The rationale for this approach is that sites having high military value are the ones most likely to be retained by the military departments. The objective function for this formulation is as follows:

$$\underset{o_s, l_{tg}}{Minimize}\ f(o_s, l_{tg}, k_{sh}) = \left(\frac{w}{u_1}\right) \times \Sigma_{s \in S}\, o_s \times nmv_s - \left(\frac{100-w}{u_2}\right) \times \Sigma_{t \in S} \Sigma_{g \in F}\, l_{tg} \times fv_{tg}/req_g$$

where

$0 \le w \le 100$	Weight parameter used to vary the emphasis between military value and functional value,
$u_1 \ge 0, u_2 \ge 0$	$u_1 = \Sigma_{s \in S}(4 - mv_s),\ u_2 = \Sigma_{f \in F} \max_{s \in S} fv_{sf}$
$nmv_s =$	$4 - mv_s$.

This formulation will be referred to as the **MINNMV** model since it minimizes the sum of $4 - mv_s$ for retained sites or activities. Site or activities having a high military value (3) will have 1 as their value. Site or activities with low military value (1) will have 3 as their value.

The parameters u_1 and u_2 are used to scale the two components of the objective function. Scaling the components of the objective function enhances the ability of the solver to find a solution. Apart from the weight parameters, these scaling parameters will scale the components of the objective function to values near 1.0 .

The weight parameter, w, can be varied to change the emphasis the formulation gives to military value versus functional value. If $w = 0$, this formulation matches the preliminary formulation (**MAXFV**) as site military value would have zero weight. Conversely, if w is set to a large value ($w = 99$), functional value would have little weight. The **MAXFV** and **MINNMV** formulations are the same formulation, only differing in the parameter w . Varying w in the formulation allows the model to be used to create a family of solutions. These points are illustrated by an example in the next section.

The component of the objective function that addresses military value of sites, $\Sigma_{s \in S}\, o_s \times nmv_s = \Sigma_{s \in S}\, o_s \times (4 - mv_s)$, affects the optimal solution as follows. (For this discussion we will ignore the functional value component of the objective function, $-\Sigma_{t \in S} \Sigma_{g \in F}\, l_{tg} \times fv_{tg}/req_g$.) If there were no constraints in the formulation, i.e., satisfy the DoD requirement, the minimum value of the objective function would be achieved by setting

$o_s = 0$ for all sites since $4 - mv_s \geq 1$ for all sites. Given that some sites have to be open, all else being equal, it is better to open a site with $mv_s = 3$ because it increases the objective function by the least amount.

The MINXCAP Formulation. If the parameter w is set to a large value ($w = 99$), this problem formulation will find the set of retained sites having the smallest total functional capacity but still able to perform the DoD functional requirement. Depending on w, functional assignments are also optimized. The objective function for this formulation is:

$$\underset{o_s,\, l_{tg},\, k_{uh}}{\text{Minimize}}\ f(o_s, l_{tg}, k_{uh}) = \left(\frac{w}{u_1}\right) \times \Sigma_{s \in S}\, o_s \times (\Sigma_{f \in F}\, cap_{sf}/req_f) - \left(\frac{100-w}{u_2}\right) \times \Sigma_{t \in S} \Sigma_{g \in F}\, l_{tg} \times fv_{tg}/req_g$$

If $w = 0$, this formulation, like the **MINNMV** formulation, is also equivalent to the **MAXFV** formulation. If w is set to a large value, excess capacity is reduced as much as possible without regard to functional values. As in the **MINNMV** formulation, u_1 and u_2 are used to scale the components of the objective function. For this formulation $u_1 = \Sigma_{s \in S} \Sigma_{f \in F}\, cap_{sf}/req_f$. The other scale parameter u_2 is set to the same value for all formulations.

The MINSITES Formulation. This formulation, depending on the value of w, will find the minimum-sized set of site or activities that can perform the DoD functional requirement. As in the previous formulations, if $w = 0$, this formulation is also equivalent to **MAXFV**. The objective function for this formulation is given by:

$$\underset{o_s,\, l_{tg},\, k_{uh}}{\text{Minimize}}\ f(o_s, l_{tg}, k_{uh}) = \left(\frac{w}{u_1}\right) \times \Sigma_{s \in S}\, o_s - \left(\frac{100-w}{u_2}\right) \times \Sigma_{t \in S} \Sigma_{g \in F}\, l_{tg} \times fv_{tg}/req_g$$

If w is set to a large value, the cross-service functional workload is assigned to the smallest possible number of sites regardless of functional values. For this formulation $u_1 = |S|$, the number of sites in the set S.

The MAXSFV formulation. This formulation maximizes the sum of the functional values for all of the retained sites. The objective function for this formulation is given by:

$$\underset{o_s,\, l_{tg},\, k_{uh}}{\text{Maximize}}\ f(o_s, l_{tg}, k_{uh}) = \left(\frac{w}{u_1}\right) \times \Sigma_{s \in S}(o_s \times \Sigma_{f \in F}\, fv_{sf}) + \left(\frac{100-w}{u_2}\right) \times \Sigma_{t \in S} \Sigma_{g \in F}\, l_{tg} \times fv_{tg}/req_g$$

For this formulation $u_1 = \Sigma_{f \in F} \Sigma_{s \in S}\, fv_{sf}$. If the number of sites to be retained is not constrained, all of the sites will be retained in the solution since the objective function is maximized when $o_s = 1$ for all sites. Obtaining meaningful results with this formulation, therefore, requires a constraint on the number of sites retained.

Policy Imperatives

A policy imperative is any statement that can be formulated as a constraint in the model. The model described here is very flexible in its capacity to handle imperatives. Examples of imperatives that can be modeled include:

- assigning functions in groups,
- increasing the average DoD military value of the sites assigned any cross-service functional workload,
- requiring the weighted functional value for a given common support function to be at least as great as some value,
- limiting the number of sites that have any cross-service functional workload assigned to them,
- requiring that each department's average military value is not allowed to go below some level,
- requiring a certain number of sites in a geographic area to remain open, and
- requiring the distribution of functional workload to follow a certain pattern, e.g., in one department, in one location, or on both coasts.

This is not an exhaustive list of the possibilities for policy imperatives. An example of a policy imperative added to the MINNMV formulation is given in the following section.

Consistent Alternatives

The functional data and constraints from all of the users may be combined into a single formulation. In the event that two users obtain solutions that are inconsistent (e.g., the solutions have a site or activity receiving cross-service functional workload in one, and losing all of its cross-service functional workload in the other) this capability can be used to resolve the inconsistency.

4. Optimization Examples

The following examples use representative, notional data to demonstrate the formulations. Three different departments, X, Y, and Z, each have 5 sites (A, B, C, D, and E). Six functions are considered: air vehicles, munitions, electronic combat, fixed-wing avionics, conventional missiles and rockets, and satellites. Table 1 shows the basic data for these sites. Table 1 also shows the DoD requirement by function and the percent of excess capacity. Percent excess capacity is calculated as

$$100 \times \left(\frac{\sum_{s \in S} cap_{sf}}{req_f} - 1 \right).$$

Preliminary Formulation (MAXFV).

Results for the MAXFV formulation are shown in table 2. If there is no functional requirement assigned to a site, the capacity for that function is shown as zero at that site even if the site has requirements for other functions assigned. Notice that, for this solution, ***all sites have some cross-service functional workload assigned.***

The column in table 2 labeled *Wgt FV* shows the weighted functional value for each function. Wgt FV for function $f \in F = \frac{\sum_{s \in S} fv_{sf} \times req_{sf}}{\sum_{s \in S} req_{sf}}$. Wgt FV is an indicator of the quality of the cross-service allocation of the functional requirement across all sites and activities. The average FV, the weighted average FV, and the weighted percent excess capacity are also shown in the table. These three numbers are gross measures of the quality of the solution.

Primary Formulation (MINNMV).

Table 3 shows the data for the optimal solution to the **MINNMV** formulation with $w = 99$. The number of sites having cross-service functional workload assigned has been reduced from 15 to six. Excess capacity is greatly reduced. The weighted percent excess capacity is only 31 percent compared to 60 for the **MAXFV** formulation. The DoD military value average is increased by 28.8 percent. The military value averages for the two departments with any sites retained have both been increased. The weighted functional value scores are not as good as the scores obtained from the **MAXFV** formulation. The average FV score is almost 14 points lower than for the **MAXFV** formulation.

Primary Formulation (MINNMV) with Policy Imperative

As an example of a policy imperative, consider the following. Suppose the user responsible for the missile function determines that only two sites should perform the conventional missiles and rockets function. The optimal solution to the original **MINNMV** formulation assigned the missile function to four different sites. Modifying the **MINNMV** formulation such that only two sites are allowed to perform the missile function results in the solution shown in table 4. The optimal solution still requires only six sites to perform the cross-service functions, but the sites are different. Only four of the sites are common to both solutions. Since the model has an additional constraint, the average military value has decreased compared to the original **MINNMV** formulation.

Parameterization of the MINNMV Formulation

Table 5 summarizes the results of varying the parameter w in the **MINNMV** formulation over the values 0, 2, 3, 5, 10, 20, 30, 40, 60, and 99 . As is to be expected, the number of sites and activities with cross-service functional workload assigned and weighted functional value decrease as w increases. The average military value generally increases as w increases. Though these results pertain only to this particular example, they clearly illustrate qualitative differences between the **MAXFV** and **MINNMV** formulations. The optimal solutions to the formulation do not change as w varies over the range of 60 to 99.

This example illustrates how the parameter w can be used to generate a family of cross-service functional solutions. For instance, a user with table 5 before him could decide that from this family of solutions, the solution obtained by setting $w = 20$ is worth exploring further since the weighted functional values are very close to the best values obtained in the **MAXFV** formulation and the weighted average percent excess capacity has been reduced from 60 to 17 percent. Table 6 displays the full output from this formulation.

Figure 1 displays this information in graphical form. The figure shows the sharp decrease in the average functional value for conventional missiles and rockets when w is changed from 20 to 30. The figure also displays the increase in average military value that is achieved by using the MINNMV formulation.

Primary Formulation (MINXCAP)

Table 7 shows the output of the MINXCAP formulation with $w = 99$. As would be expected, this formulation produces a solution that greatly reduces excess capacity, but the weighted functional values have suffered. The weighted average percent excess capacity has been reduced to almost 6 percent.

Primary Formulation (MINSITES)

The results of using the MINSITES formulation with $w = 99$ are given in table 8. The optimal solution retains only six sites. The sites are different than the sites retained in the MINNMV solution.

Primary Formulation (MAXSFV)

The results of using the MAXSFV formulation with the number of retained sites constrained to be no more than six are displayed in table 9.

Summary of Formulation Results

The following table summarizes the basic statistics for the five formulations.

Statistics	MAXFV	MINNMV	MINXCAP	MINSITES	MAXSFV
Sites retained	15	6	7	6	6
Weighted avg. percent excess capacity	60.37	31.39	6.11	12.14	24.1
Weighted average FV	84.7	73.9	74.2	76.5	62.9
Average military value	2.2	2.83	2	2.67	2.67

5. Generating Alternatives

Alternative solutions, in terms of the retained sites or activities, may be obtained by excluding a set of retained or open sites from a formulation. For example, the optimal solution obtained from the MINNMV formulation (see table 3) retains sites XA, XC, XD, ZA, ZB, and ZD. To find another optimal solution with the same objective function value or the next best solution, we define the set $\Delta_1 = \{XA, XC, XD, ZA, ZB, ZD\}$ and add the following constraints to the MINNMV formulation:

$\sum_{s \in \Delta_1} o_s \leq |\Delta_1| - \alpha$ (condition 1)

$\sum_{s \in S-\Delta_1} o_s \geq \beta$ (condition 2)

$\alpha + \beta \geq 1$

$\alpha = 0,1$ and $\beta = 0,1$.

A solution that satisfies either condition 1 ($\alpha = 1$) or condition 2 ($\beta = 1$) will be different from the original optimal solution. The formulation given above guarantees that at least one of these two conditions will hold at the optimal solution. The second best solution to the MINNMV formulation is given in table 10. The second-best solution retains sites XC, XD, YC, ZA, ZB, ZD. This solution actually has weighted functional values that are superior to those of the original optimal solution for some of the functions. Comparing values in tables 3 and 10, it would be difficult to argue that the optimal solution is clearly superior to the solution given in table 10.

If we define the set $\Delta_2 = \{XC, XD, YC, ZA, ZB, ZD\}$, then the following formulation can be used to find the third best solution:

$\sum_{s \in \Delta_1 \cap \Delta_2} o_s \leq |\Delta_1 \cap \Delta_2| - \alpha$ (condition 1)

$\sum_{s \in \Delta_1 \cap \Delta_2} o_s \geq \beta$ (condition 2)

$\left.\begin{array}{l} \sum_{s \in \Delta_1 - \Delta_2} o_s \geq \gamma \\ \sum_{s \in \Delta_2 - \Delta_1} o_s \geq \gamma \end{array}\right\rangle$ (condition 3)

$\alpha + \beta + \gamma \geq 1$

$\alpha = 0,1$, $\beta = 0,1$, and $\gamma = 0,1$.

Any solution that satisfies any one of the three conditions will be different from the first two solutions. Table 11 shows the third best solution. Comparing table 11 to tables 3 and 10 results in a less compelling case for the strength of the third best alternative. Based upon this type of comparison, the first two solutions would be subjected to further analysis before selecting one as a recommendation.

6. Optimization Software

The solutions to these optimization problems were obtained using the commercially-available, IBM Optimization Subroutine Library (OSL)[2] interfaced with AMPL[3]. The text file describing these formulations in the AMPL format is contained in appendix A. Note that all of the different objective functions are defined in this single text file. This file contains the code required to generate the second and third best alternatives. The AMPL-format data file for the

[2] *Optimization with OSL* by Ming S. Hung, Walter O. Rom, and Allan D. Waren, published by The Scientific Press.

[3] *AMPL: A Modeling Language for Mathematical Programming* by Robert Fourer, David M. Gay, and Brian Kernighan, published by The Scientific Press, 1993.

example is given in appendix B. These files are processed by the AMPL/OSL package to produce the outputs discussed in the examples section of this document.

Table 1. Joint Cross-Service Analysis Example
Basic Data

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Totals
Capacities																
Air vehicles	450	7000	2500	0	0	5000	500	0	0	0	3000	1200	0	2857	0	22,507
Munitions	850	200	4500	0	0	300	0	2000	0	0	1000	0	1000	0	0	9,850
Electronic combat	3000	0	0	0	0	1000	0	0	0	0	2000	0	0	1543	20	7,563
Fixed-wing avionics	0	0	250	3500	0	0	0	400	3500	0	1000	4000	0	2000	500	15,150
Conv. missiles/rockets	0	0	200	0	3000	0	0	200	100	2000	3000	700	200	300	200	9,900
Satelites	0	0	300	4000	0	0	0	500	0	0	250	50	0	300	2200	7,600
Function FV Scores																
Air vehicles	50	70	68	0	0	57	72	0	0	0	81	92	0	86	0	
Munitions	88	71	58	0	0	54	0	88	0	0	72	0	75	0	0	
Electronic combat	67	0	0	0	0	91	0	0	0	0	52	0	0	78	77	
Fixed-wing avionics	0	0	92	94	0	0	0	78	69	0	72	93	0	66	71	
Conv. missiles/rockets	0	0	62	0	89	0	0	59	93	92	56	59	50	65	91	
Satelites	0	0	71	58	0	0	0	64	0	0	85	61	0	73	93	
Department Military Value	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1	

Function	DoD req.	Pct. excess
Air vehicles	9,463	137.8
Munitions	5,503	79.0
Electronic combat	3,234	133.9
Fixed-wing avionics	3,775	301.3
Conv. missiles/rockets	3,743	164.5
Satelites	2,480	206.5

Table 2. MAXFV Model Output

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	15	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	7000	0	0	0	0	500	0	0	0	3000	1200	0	2857	0	14557	53.8
Munitions	850	200	4500	0	0	0	0	2000	0	0	1000	0	1000	0	0	9550	73.5
Electronic combat	3000	0	0	0	0	1000	0	0	0	0	0	0	0	1543	20	5563	72.0
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	4000	0	0	0	7500	98.7
Conv. missiles/rockets	0	0	0	0	3000	0	0	0	100	2000	0	0	0	0	200	5300	41.6
Satelites	0	0	0	0	0	0	0	0	0	0	250	0	0	300	2200	2750	10.9
																Wgt. avg.	60.37
Workload assigned																Totals	
Air vehicles	0	1908	0	0	0	0	500	0	0	0	3000	1200	0	2857	0	9463	
Munitions	850	200	453	0	0	0	0	2000	0	0	1000	0	1000	0	0	5503	
Electronic combat	671	0	0	0	0	1000	0	0	0	0	0	0	0	1543	20	3234	
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	275	0	0	0	3775	
Conv. missiles/rockets	0	0	0	0	1443	0	0	0	100	2000	0	0	0	0	200	3743	
Satelites	0	0	0	0	0	0	0	0	0	0	250	0	0	30	2200	2480	
Department avg. MV			2.4					1.8					2.4				
Percent change			-0.0					0.0					-0.0				

DoD average MV	2.20
Percent change	0.0

DoD weighted FVs	
Function	Wgt FV
Air vehicles	81.2
Munitions	79.6
Electronic combat	79.7
Fixed-wing avionics	93.9
Conv. missiles/rockets	90.8
Satelites	92.0
Average FV	86.2
Weighted avg. FV	84.7

Table 3. MINNMV Model Output

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	1	0	1	1	0	0	0	0	0	0	1	1	0	1	0	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	0	2500	0	0	0	0	0	0	0	3000	1200	0	2857	0	9557	1.0
Munitions	850	0	4500	0	0	0	0	0	0	0	1000	0	0	0	0	6350	15.4
Electronic combat	3000	0	0	0	0	0	0	0	0	0	0	0	0	1543	0	4543	40.5
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	4000	0	0	0	7500	98.7
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	3000	700	0	300	0	4200	12.2
Satelites	0	0	300	4000	0	0	0	0	0	0	250	50	0	300	0	4900	97.6
																Wgt. avg.	31.39
Workload assigned																Totals	
Air vehicles	0	0	2406	0	0	0	0	0	0	0	3000	1200	0	2857	0	9463	
Munitions	850	0	3653	0	0	0	0	0	0	0	1000	0	0	0	0	5503	
Electronic combat	1691	0	0	0	0	0	0	0	0	0	0	0	0	1543	0	3234	
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	275	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	2543	700	0	300	0	3743	
Satelites	0	0	300	1580	0	0	0	0	0	0	250	50	0	300	0	2480	
Department avg. MV			2.7					0.0					3.0				
Percent change			11.1					-100.0					25.0				

DoD average MV	2.83
Percent change	28.8

DoD weighted FVs Function	Wgt FV
Air vehicles	80.6
Munitions	65.2
Electronic combat	72.2
Fixed-wing avionics	93.9
Conv. missiles/rockets	57.6
Satelites	64.2
Average FV	72.3
Weighted avg. FV	73.9

Table 4. MINNMV Model with Policy Imerative Output

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	7000	0	0	0	0	0	0	0	0	3000	0	0	2857	0	12857	35.9
Munitions	0	200	4500	0	0	0	0	0	0	0	1000	0	0	0	0	5700	3.6
Electronic combat	0	0	0	0	0	0	0	0	0	0	2000	0	0	1543	0	3543	9.6
Fixed-wing avionics	0	0	250	3500	0	0	0	0	0	0	1000	0	0	0	0	4750	25.8
Conv. missiles/rockets	0	0	0	0	3000	0	0	0	0	0	3000	0	0	0	0	6000	60.3
Satelites	0	0	300	4000	0	0	0	0	0	0	250	0	0	300	0	4850	95.6
																Wgt. avg.	33.70
Workload assigned																**Totals**	
Air vehicles	0	3606	0	0	0	0	0	0	0	0	3000	0	0	2857	0	9463	
Munitions	0	200	4303	0	0	0	0	0	0	0	1000	0	0	0	0	5503	
Electronic combat	0	0	0	0	0	0	0	0	0	0	1691	0	0	1543	0	3234	
Fixed-wing avionics	0	0	250	3500	0	0	0	0	0	0	25	0	0	0	0	3775	
Conv. missiles/rockets	0	0	0	0	3000	0	0	0	0	0	743	0	0	0	0	3743	
Satelites	0	0	300	1630	0	0	0	0	0	0	250	0	0	300	0	2480	
Department avg. MV			2.3					0.0					3.0				
Percent change			-8.3					-100.0					25.0				

DoD average MV	2.50
Percent change	13.6

DoD weighted FVs

Function	Wgt FV
Air vehicles	78.3
Munitions	61.0
Electronic combat	64.4
Fixed-wing avionics	93.7
Conv. missiles/rockets	82.4
Satelites	64.1
Average FV	74.0
Weighted avg. FV	74.7

Table 5. Parameterization of the MINNMV Model

	Percent of weight on FV									
	0 MAXFV	2	3	5	10	20	30	40	60	99 MINNMV
Sites/activities open	15	13	12	11	9	8	7	6	6	6
Percent excess										
Air vehicles	53.8	48.5	48.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Munitions	73.5	73.5	73.5	69.9	51.7	51.7	51.7	15.4	15.4	15.4
Electronic combat	72.0	72.0	72.0	72.0	72.0	41.1	41.1	41.1	40.5	40.5
Fixed-wing avionics	98.7	98.7	6.0	6.0	6.0	6.0	6.0	6.0	98.7	98.7
Conv. missiles/rockets	41.6	38.9	38.9	38.9	4.2	4.2	22.9	17.6	12.2	12.2
Satelites	10.9	10.9	10.9	10.9	10.9	10.9	10.9	10.9	97.6	97.6
Wgt. avg. % excess	**60.37**	**58.24**	**45.83**	**29.16**	**21.00**	**17.46**	**19.94**	**12.14**	**31.39**	**31.39**
Weighted FV										
Air vehicles	81.2	81.1	81.1	80.6	80.6	80.6	80.6	80.6	80.6	80.6
Munitions	79.6	79.6	79.6	79.2	76.1	76.1	76.1	65.2	65.2	65.2
Electronic combat	79.7	79.7	79.7	79.7	79.7	72.3	72.3	72.3	72.2	72.2
Fixed-wing avionics	93.9	93.9	93.0	93.0	93.0	93.0	93.0	93.0	93.9	93.9
Conv. missiles/rockets	90.8	90.7	90.7	90.7	85.4	85.4	59.6	59.5	57.6	57.6
Satelites	92.0	92.0	92.0	92.0	92.0	92.0	92.0	92.0	64.2	64.2
Average FV	**86.2**	**86.2**	**86.0**	**85.9**	**84.5**	**83.2**	**78.9**	**77.1**	**72.3**	**72.3**
Weighted avg. FV	**84.7**	**84.6**	**84.5**	**84.2**	**82.9**	**82.1**	**78.6**	**76.5**	**73.9**	**73.9**
DoD average MV	**2.20**	**2.31**	**2.33**	**2.27**	**2.44**	**2.50**	**2.71**	**2.67**	**2.83**	**2.83**

Figure 1. Parameterization of MINNMV

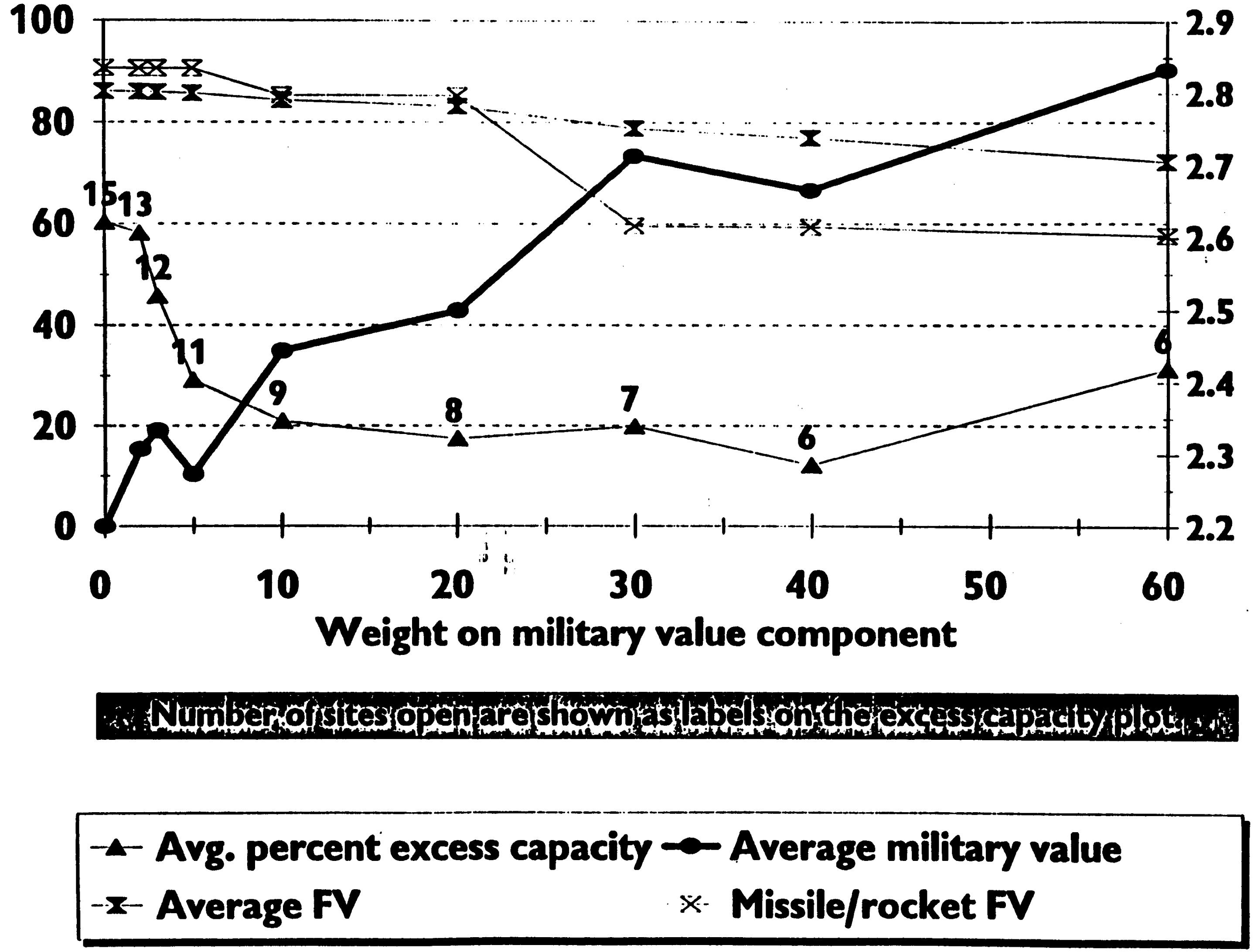


Table 6. MINNMV Model Output with Weight = 20

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	1	0	1	0	1	0	0	1	0	0	1	1	0	1	1	8	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	0	2500	0	0	0	0	0	0	0	3000	1200	0	2857	0	9557	1.0
Munitions	850	0	4500	0	0	0	0	2000	0	0	1000	0	0	0	0	8350	51.7
Electronic combat	3000	0	0	0	0	0	0	0	0	0	0	0	0	1543	20	4563	41.1
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	4000	6.0
Conv. missiles/rockets	0	0	200	0	3000	0	0	200	0	0	0	0	0	300	200	3900	4.2
Satelites	0	0	0	0	0	0	0	0	0	0	250	0	0	300	2200	2750	10.9
																Wgt. avg.	17.46
Workload assigned																Totals	
Air vehicles	0	0	2406	0	0	0	0	0	0	0	3000	1200	0	2857	0	9463	
Munitions	850	0	1653	0	0	0	0	2000	0	0	1000	0	0	0	0	5503	
Electronic combat	1671	0	0	0	0	0	0	0	0	0	0	0	0	1543	20	3234	
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	3775	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	3000	0	0	43	0	0	0	0	0	300	200	3743	
Satelites	0	0	0	0	0	0	0	0	0	0	250	0	0	30	2200	2480	
Department avg. MV			2.3					3.0					2.5				
Percent change			-2.8					66.7					4.2				
DoD average MV								2.50									
Percent change								13.6									

DoD weighted FVs	
Function	Wgt FV
Air vehicles	80.6
Munitions	76.1
Electronic combat	72.3
Fixed-wing avionics	93.0
Conv. missiles/rockets	85.4
Satelites	92.0
Average FV	83.2
Weighted avg. FV	82.1

Table 7. MINXCAP Model Output

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	1	0	1	0	1	1	1	0	0	0	0	1	0	0	1	7	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	450	0	2500	0	0	5000	500	0	0	0	0	1200	0	0	0	9650	2.0
Munitions	850	0	4500	0	0	300	0	0	0	0	0	0	0	0	0	5650	2.7
Electronic combat	3000	0	0	0	0	1000	0	0	0	0	0	0	0	0	20	4020	24.3
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	4000	6.0
Conv. missiles/rockets	0	0	200	0	3000	0	0	0	0	0	0	700	0	0	200	4100	9.5
Satelites	0	0	300	0	0	0	0	0	0	0	0	0	0	0	2200	2500	0.8
																Wgt. avg.	6.11
Workload assigned																Totals	
Air vehicles	263	0	2500	0	0	5000	500	0	0	0	0	1200	0	0	0	9463	
Munitions	850	0	4500	0	0	153	0	0	0	0	0	0	0	0	0	5503	
Electronic combat	2214	0	0	0	0	1000	0	0	0	0	0	0	0	0	20	3234	
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	3775	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	3000	0	0	0	0	0	0	343	0	0	200	3743	
Satelites	0	0	280	0	0	0	0	0	0	0	0	0	0	0	2200	2480	
Department avg. MV			2.3					1.5					2.0				
Percent change			-2.8					-16.7					-16.7				
DoD average MV								2.00									
Percent change								-9.1									

DoD weighted FVs	
Function	Wgt FV
Air vehicles	64.9
Munitions	62.5
Electronic combat	74.5
Fixed-wing avionics	93.0
Conv. missiles/rockets	84.9
Satelites	90.5
Average FV	78.4
Weighted avg. FV	74.2

Table 8. MINSITES Model Output

Function	Department X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	1	0	1	0	0	0	0	0	0	0	1	1	0	1	1	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	0	2500	0	0	0	0	0	0	0	3000	1200	0	2857	0	9557	1.0
Munitions	850	0	4500	0	0	0	0	0	0	0	1000	0	0	0	0	6350	15.4
Electronic combat	3000	0	0	0	0	0	0	0	0	0	0	0	0	1543	20	4563	41.1
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	4000	6.0
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	3000	700	0	300	200	4400	17.6
Satellites	0	0	0	0	0	0	0	0	0	0	250	0	0	300	2200	2750	10.9
																Wgt. avg.	12.14
Workload assigned																**Totals**	
Air vehicles	0	0	2406	0	0	0	0	0	0	0	3000	1200	0	2857	0	9463	
Munitions	850	0	3653	0	0	0	0	0	0	0	1000	0	0	0	0	5503	
Electronic combat	1671	0	0	0	0	0	0	0	0	0	0	0	0	1543	20	3234	
Fixed-wing avionics	0	0	0	0	0	0	0	0	0	0	0	3775	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	2343	700	0	300	200	3743	
Satellites	0	0	0	0	0	0	0	0	0	0	250	0	0	30	2200	2480	
Department avg. MV			3.0					0.0					2.5				
Percent change			25.0					-100.0					4.2				

DoD average MV	2.67
Percent change	21.2

DoD weighted FVs	
Function	Wgt FV
Air vehicles	80.6
Munitions	65.2
Electronic combat	72.3
Fixed-wing avionics	93.0
Conv. missiles/rockets	59.5
Satellites	92.0
Average FV	77.1
Weighted avg. FV	76.5

Table 9. MAXSFV Model Output

Function	Department X: A	X: B	X: C	X: D	X: E	Y: A	Y: B	Y: C	Y: D	Y: E	Z: A	Z: B	Z: C	Z: D	Z: E	Retained totals	Percent excess
Retain=1, Close=0	0	0	1	1	0	1	0	0	0	0	1	1	0	1	0	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	0	2500	0	0	5000	0	0	0	0	3000	0	0	0	0	10500	11.0
Munitions	0	0	4500	0	0	300	0	0	0	0	1000	0	0	0	0	5800	5.4
Electronic combat	0	0	0	0	0	0	0	0	0	0	2000	0	0	1543	0	3543	9.6
Fixed-wing avionics	0	0	250	0	0	0	0	0	0	0	1000	4000	0	2000	0	7250	92.1
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	3000	700	0	0	0	3900	4.2
Satelites	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	4000	61.3
																Wgt. avg.	24.10
Workload assigned																Totals	
Air vehicles	0	0	2500	0	0	5000	0	0	0	0	1963	0	0	0	0	9463	
Munitions	0	0	4500	0	0	300	0	0	0	0	703	0	0	0	0	5503	
Electronic combat	0	0	0	0	0	0	0	0	0	0	2000	0	0	1234	0	3234	
Fixed-wing avionics	0	0	250	0	0	0	0	0	0	0	1000	525	0	2000	0	3775	
Conv. missiles/rockets	0	0	43	0	0	0	0	0	0	0	3000	700	0	0	0	3743	
Satelites	0	0	0	2480	0	0	0	0	0	0	0	0	0	0	0	2480	

Function	X	Y	Z
Department avg. MV	2.5	2.0	3.0
Percent change	4.2	11.1	25.0

DoD average MV	2.67
Percent change	21.2

DoD weighted FVs	
Function	Wgt FV
Air vehicles	64.9
Munitions	59.6
Electronic combat	61.9
Fixed-wing avionics	73.1
Conv. missiles/rockets	56.6
Satelites	58.0
Average FV	62.3
Weighted avg. FV	62.9

Table 10. MINNMV Model Output: Alternative 1

Function	X A	X B	X C	X D	X E	Y A	Y B	Y C	Y D	Y E	Z A	Z B	Z C	Z D	Z E	Retained totals	Percent excess
Retain=1, Close=0	0	0	1	1	0	0	0	1	0	0	1	1	0	1	0	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	0	2500	0	0	0	0	0	0	0	3000	1200	0	2857	0	9557	1.0
Munitions	0	0	4500	0	0	0	0	2000	0	0	1000	0	0	0	0	7500	36.3
Electronic combat	0	0	0	0	0	0	0	0	0	0	2000	0	0	1543	0	3543	9.6
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	4000	0	0	0	7500	98.7
Conv. missiles/rockets	0	0	200	0	0	0	0	200	0	0	3000	700	0	300	0	4400	17.6
Satelites	0	0	300	4000	0	0	0	500	0	0	250	50	0	300	0	5400	117.7
																Wgt. avg.	34.41
Workload assigned																Totals	
Air vehicles	0	0	2406	0	0	0	0	0	0	0	3000	1200	0	2857	0	9463	
Munitions	0	0	2503	0	0	0	0	2000	0	0	1000	0	0	0	0	5503	
Electronic combat	0	0	0	0	0	0	0	0	0	0	1691	0	0	1543	0	3234	
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	275	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	0	0	0	200	0	0	2343	700	0	300	0	3743	
Satelites	0	0	300	1080	0	0	0	500	0	0	250	50	0	300	0	2480	
Department avg. MV			2.5					3.0					3.0				
Percent change			4.2					66.7					25.0				

DoD average MV	2.83
Percent change	28.8

Function	Wgt FV
Air vehicles	80.6
Munitions	71.4
Electronic combat	64.4
Fixed-wing avionics	93.9
Conv. missiles/rockets	57.8
Satelites	65.4
Average FV	72.3
Weighted avg. FV	74.4

(Table title: DoD weighted FVs)

Table 11. MINNMV Model Output: Alternative 2

Function	Department															Retained totals	Percent excess
	X					Y					Z						
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E		
Retain=1, Close=0	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	6	
Department Mil. Val.	3	3	3	2	1	2	1	3	2	1	3	3	2	3	1		
Capacities																	
Air vehicles	0	7000	0	0	0	0	0	0	0	0	3000	1200	0	0	0	11200	18.4
Munitions	850	200	4500	0	0	0	0	0	0	0	1000	0	0	0	0	6550	19.0
Electronic combat	3000	0	0	0	0	0	0	0	0	0	2000	0	0	0	0	5000	54.6
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	4000	0	0	0	7500	98.7
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	3000	700	0	0	0	3900	4.2
Satellites	0	0	300	4000	0	0	0	0	0	0	250	50	0	0	0	4600	85.5
																Wgt. avg.	37.42
Workload assigned																**Totals**	
Air vehicles	0	5263	0	0	0	0	0	0	0	0	3000	1200	0	0	0	9463	
Munitions	850	200	3453	0	0	0	0	0	0	0	1000	0	0	0	0	5503	
Electronic combat	3000	0	0	0	0	0	0	0	0	0	234	0	0	0	0	3234	
Fixed-wing avionics	0	0	0	3500	0	0	0	0	0	0	0	275	0	0	0	3775	
Conv. missiles/rockets	0	0	200	0	0	0	0	0	0	0	2843	700	0	0	0	3743	
Satellites	0	0	300	1880	0	0	0	0	0	0	250	50	0	0	0	2480	
Department avg. MV			2.8					0.0					3.0				
Percent change			14.6					-100.0					25.0				

DoD average MV	2.83
Percent change	28.8

DoD weighted FVs	
Function	Wgt FV
Air vehicles	76.3
Munitions	65.7
Electronic combat	65.9
Fixed-wing avionics	93.9
Conv. missiles/rockets	56.9
Satellites	62.4
Average FV	70.2
Weighted avg. FV	71.6

Appendix A

AMPL Model Input File

```
# JCSG Model Example

# Ronald H. Nickel, Ph.D.
# LTC Roy Rice, USAF

# 8-3-94

set X_sites;         # The set of Department X sites.
set Y_sites;         # The set of Department Y sites.
set Z_sites;         # The set of Department Z sites.

set SITE := X_sites union {Y_sites union Z_sites};
            # The set of all labs and T&E sites.

set EXCLD1 within SITE default {}; # A solution to be excluded.

set EXCLD2 within SITE default {}; # A solution to be excluded.

set EXCLD_INTER := if card(EXCLD2) > 0 then (EXCLD1 inter EXCLD2)
                        else EXCLD1;

set EXCLD_1DIFF2 := EXCLD1 diff EXCLD2;    # Sites in EXCLD1 but not
                                           # in EXCLD2.

set EXCLD_2DIFF1 := EXCLD2 diff EXCLD1;    # Sites in EXCLD2 but not
                                           # in EXCLD1.

set EXCLD_COMPLEMENT := SITE diff (EXCLD1 union EXCLD2);
              # The set of sites not in EXCLD1 or EXCLD2.

param excld_num := max(0,card(EXCLD_INTER)-1);

set FUNC;        # The set of functions.

set SITE_CAP within {SITE, FUNC} ;  # The set of site/function
                        # combinations that are
                        # meaningful.

param CAPAC {SITE_CAP}; # The functional capacity at each site for each
              # meaningful site/function combination.

param no_func := card(FUNC);     # The number of function types.

# Define the set performing missile functions.

set MISSLE_FUNC within {FUNC};

param missile_sites >= 0, default 15;
            # Number of sites allowed to perform the
            # missile function.  Used in the policy
            # imperative example (missile_sites = 3).

param max_sites >= 0, default card(SITE);
            # Number of open sites allowed in the
            # solution.

param REQ {FUNC};   # The DoD requirement for each function.
```

```
param MV {SITE};     # Military value for each site.

param NMV {s in SITE} := 4 - MV[s]; # Negative MV scoring.

param FV {SITE_CAP} >= 0.0; # Functional value by site and function.

param min_assign default 0.001; # Cannot assign less than
                                # min_assign * CAPAC[s,f] of
                                # function f to site s.

#
# Calculate upper bounds for the objective function components.
#

param MINNMV_UB := sum {s in SITE} NMV[s];

param MINSITES_UB := card(SITE);

param MINXCAP_UB := sum {(s,f) in SITE_CAP} CAPAC[s,f]/REQ[f];

param MAXSFV_UB := sum {(s,f) in SITE_CAP} FV[s,f];

param MAXFV_UB := sum {f in FUNC} max {(s,f) in SITE_CAP} FV[s,f];

#
# Use WGT_PCT to weight the functional value and non-functional value
# components of the objective functions.
#

param WGT_PCT >= 0, <= 100, default 99; # Percent of weight to put on
        # non-functional-value portion of the objective function.

param WGT1 := WGT_PCT;  # Weight for non-FV portion of the objective
                # functions.

param WGT2 := 100-WGT1; # Weight for FV portion of the objective functions.

#
# Decision variables
#

var OPEN {SITE} binary >= 0;     # Open or closed decision variable for
                # each site.

var SITE_LOAD {(s,f) in SITE_CAP} >= 0.0, <= CAPAC[s,f];
            # Amount of the requirement for function f to
            # be assigned to site s . Amount assigned
            # is limited by capacity of site s to perform
            # function f.

var SITE_FUNC {(s,f) in SITE_CAP} binary;
            # 1 if any assignment of workload for function
            # f is made to site s; 0 otherwise.

# The following variables, ALPHA, BETA,and GAMMA, are used to find
# alternative solutions.
```

```
var ALPHA binary; # At least one site from the intersection is excluded
                  # from the solution.

var BETA binary;  # At least one site from the complement of the union
                  # is included is included in the solution.

var GAMMA binary; # At least one site from
                  #    EXCLD1 - (EXCLD1 intersect EXCLD2)
                  # and at least one site from
                  #    EXCLD2 - (EXCLD1 intersect EXCLD2)
                  # are included in the solution.

#
# Objective Functions.
#

# Minimize total open site negative military value and
# maximize the normalized FV-weighted assignment of functional workload
# to sites.

minimize MINNMV:
     (WGT1/MINNMV_UB) * sum {s in SITE} OPEN[s]*NMV[s]
     - (WGT2/MAXFV_UB) * sum {(t,g) in SITE_CAP} FV[t,g]
     * (SITE_LOAD[t,g]/REQ[g]);

# Minimize the number of open sites and maximize the normalized
# FV-weighted assignment of functional workload to sites.

minimize MINSITES:
     (WGT1/MINSITES_UB) * sum {s in SITE} OPEN[s]
     - (WGT2/MAXFV_UB) * sum {(t,g) in SITE_CAP} FV[t,g]
     * (SITE_LOAD[t,g]/REQ[g]);

# Minimize total capacity and maximize the normalized FV-weighted
# assignment of functional workload to sites.

minimize MINXCAP:
     (WGT1/MINXCAP_UB) * sum {s in SITE}  OPEN[s] *
         (sum {(s,f) in SITE_CAP} CAPAC[s,f]/REQ[f])
     - (WGT2/MAXFV_UB) * sum {(t,g) in SITE_CAP} FV[t,g]
     * (SITE_LOAD[t,g]/REQ[g]);

# Maximize functional value without workload assignment weightings
# and maximize the normalized FV-weighted assignment of functional
# workload to sites.

maximize MAXSFV:
     (WGT1/MAXSFV_UB) * sum {(s,f) in SITE_CAP} FV[s,f]
     - (WGT2/MAXFV_UB) * sum {(t,g) in SITE_CAP} FV[t,g]
     * (SITE_LOAD[t,g]/REQ[g]);

#
# Constraints
#

# The requirement for each function has to be met.
```

```
subject to func_assgn {f in FUNC}:
    sum {(s,f) in SITE_CAP} SITE_LOAD[s,f] = REQ[f];

# Cannot assign functional workload to a site unless
# the site is open for assignment of that function.

subject to func_open {(s,f) in SITE_CAP}:
    SITE_LOAD[s,f] <= SITE_FUNC[s,f]*CAPAC[s,f];

# Sites with no functional requirement assigned
# are closed.

subject to site_closed {s in SITE}:
    OPEN[s] <= sum {(s,f) in SITE_CAP} SITE_FUNC[s,f];

# Allocation of functional requirements cannot be made
# to sites that are not open.

subject to site_open {s in SITE}:
    sum {(s,f) in SITE_CAP} SITE_FUNC[s,f] <= OPEN[s] * no_func;

# SITE_FUNC variables are set to 0 if little or no functional
# workload is assigned to a site.

subject to site_func_0 {(s,f) in SITE_CAP}:
    SITE_FUNC[s,f] <= SITE_LOAD[s,f]/(min_assign * CAPAC[s,f]);

# This constraint is an example of a policy imperative.
# Constrain the number of sites doing munitions work.
# This constraint only constrains the model if
#
#   missile_sites < card(SITE).

subject to missile_2 {f in MISSLE_FUNC}:
    sum {(s,f) in SITE_CAP} SITE_FUNC[s,f] <= missile_sites;

# This constraint is used to constrain the number of
# open sites in a solution.  max_sites has a default
# value equal to card(SITE), i.e., it does not constrain
# the solution unless max_sites is set to a lower value.

subject to no_sites:
    sum {s in SITE} OPEN[s] <= max_sites;


#
# Exclude solutions defined by the sets EXCLD1 and EXCLD2.
#

subject to alt_opt_cond_1:
    sum {s in EXCLD_INTER} OPEN[s] <= excld_num + 1 - ALPHA;

subject to alt_opt_cond_2:
   sum {s in EXCLD_COMPLEMENT} OPEN[s] >= BETA;

subject to alt_opt_cond_3a:
   sum {s in EXCLD_1DIFF2} OPEN[s] >= GAMMA;
```

```
subject to alt_opt_cond_3b:
   sum {s in EXCLD_2DIFF1} OPEN[s] >= GAMMA;

subject to alt_opt_cond_123:
   ALPHA + BETA + GAMMA >= 1;
```

Appendix B

AMPL Data Input File

```
# Data file for JCSG optimization examples.

# Ron NIckel
# 7-6-94

set X_sites :=
    X_A
    X_B
    X_C
    X_D
    X_E;

set Y_sites :=
    Y_A
    Y_B
    Y_C
    Y_D
    Y_E;

set Z_sites :=
    Z_A
    Z_B
    Z_C
    Z_D
    Z_E;

set EXCLD1 := X_A X_C X_D Z_A Z_B Z_D;

set EXCLD2 := X_C X_D Y_C Z_A Z_B Z_D;

set FUNC :=
    Air_Veh
    Mun
    E_Cmbt
    Avion
    Mis
    Sat;

set SITE_CAP :  Air_Veh Mun    E_Cmbt  Avion   Mis     Sat :=
        X_A             +               +       +               -       -
        X_B             +               +       -               -       -
        X_C             +               +       -               +       +
        X_D             -               -       -               +       -
        X_E             -               -       -               -       +
        Y_A             +               +       +               -       -
        Y_B             +               -       -               -       -
        Y_C             -               +       -               +       +
        Y_D             -               -       -               +       +
        Y_E             -               -       -               -       +
        Z_A             +               +       +               +       +
        Z_B             +               -       -               +       +
        Z_C             -               +       -               -       +
        Z_D             +               -       +               +       +
        Z_E             -               -       +               +       +

# Used to model the policy imperative.
```

```
set MISSLE_FUNC := Mis;

param CAPAC:    Air_Veh Mun     E_Cmbt  Avion   Mis     Sat :=
        X_A     450             850     3000            .       .       .
        X_B     7000            200     .               .       .       .
        X_C     2500            4500    .               250     200     300
        X_D     .               .       .               3500    .       4000
        X_E     .               .       .               .       3000    .
        Y_A     5000            300     1000            .       .       .
        Y_B     500             .       .               .       .       .
        Y_C     .               2000    .               400     200     500
        Y_D     .               .       .               3500    100     .
        Y_E     .               .       .               .       2000    .
        Z_A     3000            1000    2000            1000    3000    250
        Z_B     1200            .       .               4000    700     50
        Z_C     .               1000    .               .       200     .
        Z_D     2857            .       1543            2000    300     300
        Z_E     .               .       20              500     200     2200;

param FV:       Air_Veh Mun     E_Cmbt  Avion   Mis     Sat :=
    X_A     50              88      67              .       .       .
    X_B     70              71      .               .       .       .
    X_C     68              58      .               92      62      71
    X_D     .               .       .               94      .       58
    X_E     .               .       .               .       89      .
    Y_A     57              54      91              .       .       .
    Y_B     72              .       .               .       .       .
    Y_C     .               88      .               78      59      64
    Y_D     .               .       .               69      93      .
    Y_E     .               .       .               .       92      .
    Z_A     81              72      52              72      56      85
    Z_B     92              .       .               93      59      61
    Z_C     .               75      .               .       50      .
    Z_D     86              .       78              66      65      73
    Z_E     .               .       77              71      91      93;

param REQ :=
    Air_Veh 9463
    Mun             5503
    E_Cmbt  3234
    Avion           3775
    Mis             3743
    Sat             2480;

# Banded military values for each site.
# 3 is good, 1 is bad.

param MV :=
    X_A     3
    X_B     3
    X_C     3
    X_D     2
    X_E     1
    Y_A     2
    Y_B     1
    Y_C     3
    Y_D     2
```

```
Y_E      1
Z_A      3
Z_B      3
Z_C      2
Z_D      3
Z_E      1;
```